

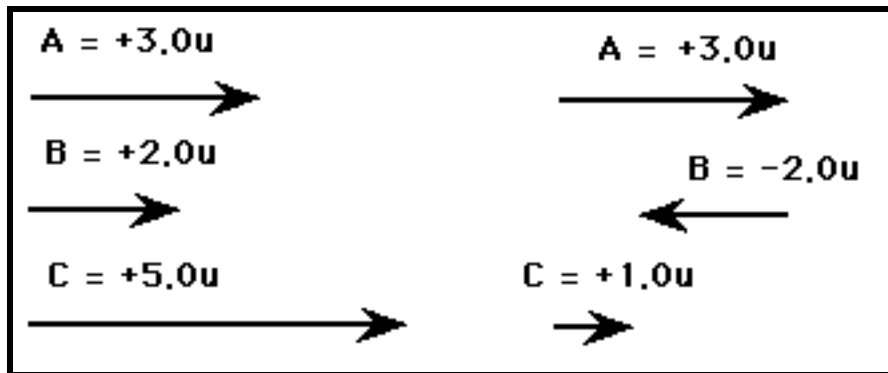
## Unit IV: Introduction to Vector Analysis

As you learned in the last unit, there is a difference between speed and velocity. Speed is an example of a **scalar**: a quantity that has only magnitude. Velocity is an example of a **vector**: a quantity that has both magnitude and direction. Velocity is a more useful description of an object's motion than just the speed because the direction of motion is often important to know.

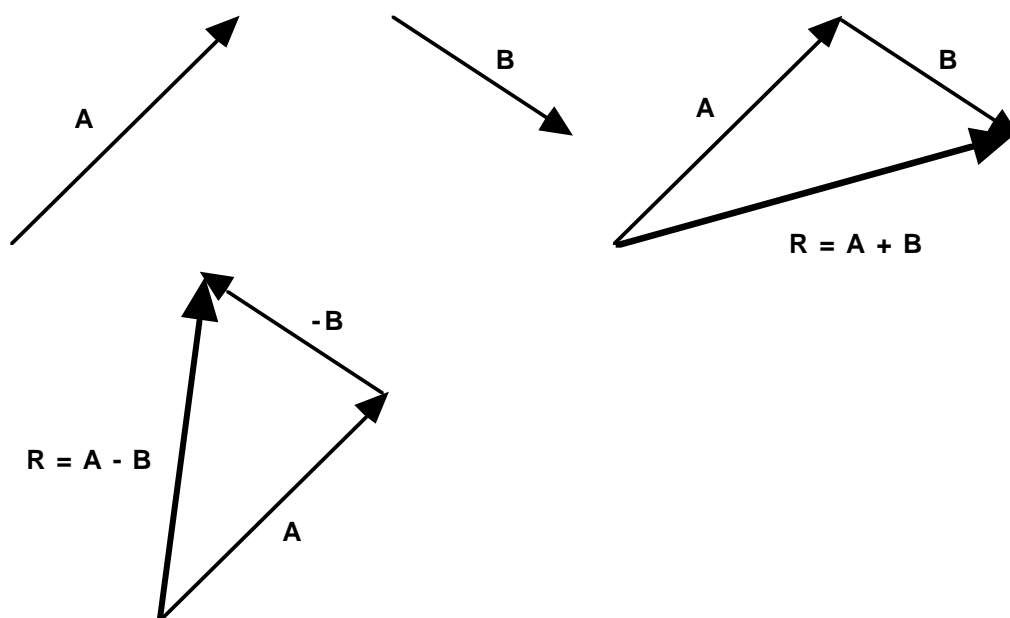
For example, you know that if your plane is heading north on a windy day, you want to know which way as well as how hard the wind is blowing. To handle this sort of problem we need to learn how to work with vectors.

### Graphical Method

When you **add** two vectors graphically, you simply move vector B so that its tail is placed at the tip of vector A, then draw the resultant vector from the tail of A to the tip of B. Subtraction is accomplished by simply *adding the opposite* of the vector. In the most trivial case of vector addition, suppose that A represents your velocity and B represents the wind velocity. Your resulting velocity (C) could be as high as  $+5u$  or as low as  $+1u$ .



While this method works well in simple cases, it is often difficult to determine the magnitude of the resultant unless you work carefully on graph paper. The example below shows how you add or subtract vectors which are **not** parallel.

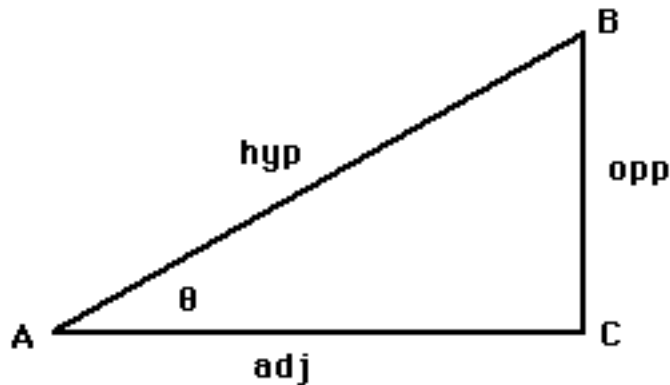


As you can see, because the vectors are not parallel, you cannot simply add the magnitudes of **A** and **B** to find the magnitude of **R**.

In this course we will use a trigonometric approach to adding vectors. Before we learn how to add vectors by breaking them up into their x and y components, it would be useful to review some basic trigonometry.

### Basic Trig Review

Consider the right triangle below. You certainly have learned the Pythagorean formula which relates the length of the hypotenuse to the lengths of the other two sides.  $hyp^2 = opp^2 + adj^2$  You should be familiar with the row of trig formulas below which relate two sides of the triangle to the angle  $\theta$  (theta). You may have less experience with the inverse functions. These are what you use when the sides are known, but the angle is not.



$$\sin \theta = \frac{opp}{hyp} \qquad \cos \theta = \frac{adj}{hyp} \qquad \tan \theta = \frac{opp}{adj}$$

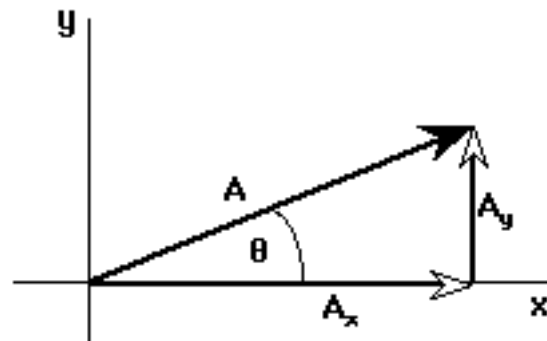
$$\sin^{-1}\left(\frac{opp}{hyp}\right) = \theta \qquad \cos^{-1}\left(\frac{adj}{hyp}\right) = \theta \qquad \tan^{-1}\left(\frac{opp}{adj}\right) = \theta$$

The reason we work with right triangles is that the x and y axes are perpendicular. The vector A below can be thought of as the sum of other two vectors,  $A_x$  and  $A_y$ . It is possible to break up any vector into its x and y components. Consider the example below.

$$\cos \theta = \frac{A_x}{A} \qquad \sin \theta = \frac{A_y}{A}$$

$$A_x = A \cos \theta \qquad A_y = A \sin \theta$$

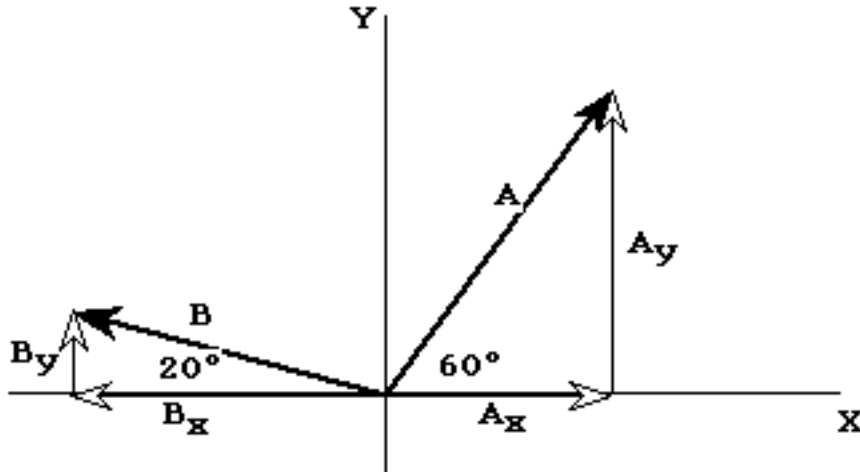
$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



Once you have resolved vectors into their x and y components, you can add the x-components of the vectors to find the x-component of the resultant. Likewise, you can do the same for the y-components. Once you have the x and y components of the resultant R, you can use the Pythagorean formula to determine the length of R.

Consider the example below:

Vector A is 10 units long, while vector B has a length of 7 units.



As you can see above, vectors **A** and **B** have been broken into their x and y components.

$$\begin{array}{llll}
 A_x = A \cos 60^\circ & A_y = A \sin 60^\circ & B_x = B \cos 20^\circ & B_y = B \sin 20^\circ \\
 A_x = 10(0.50) & A_y = 10(0.866) & B_x = 7(0.94) & B_y = 7(0.342) \\
 A_x = 5.0 & A_y = 8.7 & B_x = -6.6 & B_y = 2.4
 \end{array}$$

Note that **B<sub>x</sub>** is negative; this is due to the fact that x is (-) in the 2nd and 3rd quadrants.

vector	x-comp	y-comp
<b>A</b>	<b>5.0</b>	<b>8.7</b>
<b>B</b>	<b>-6.6</b>	<b>2.4</b>
<b>R</b>	<b>-1.6</b>	<b>11.1</b>

The length of the resultant vector **R** is computed at right.

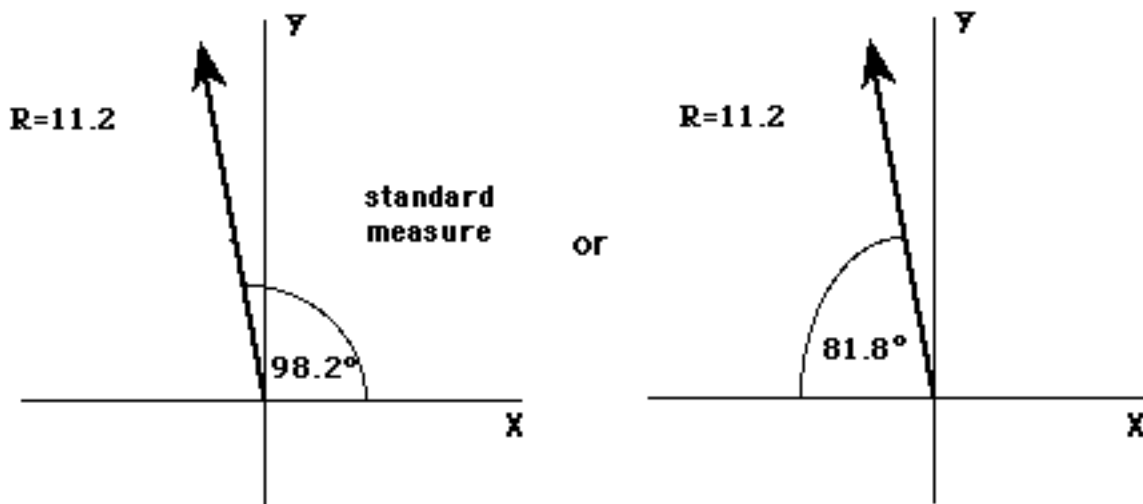
$$\begin{aligned}
 R^2 &= R_x^2 + R_y^2 \\
 R &= \sqrt{(-1.6)^2 + (11.1)^2} \\
 R &= 11.2
 \end{aligned}$$

To determine the angle the resultant makes with the x-axis, you must use the inverse tan function and some common sense.

$$\begin{aligned}
 \theta &= \tan^{-1}\left(\frac{R_y}{R_x}\right) \\
 \theta &= \tan^{-1}\left(\frac{11.1}{-1.6}\right) \\
 \theta &= -81.8^\circ
 \end{aligned}$$

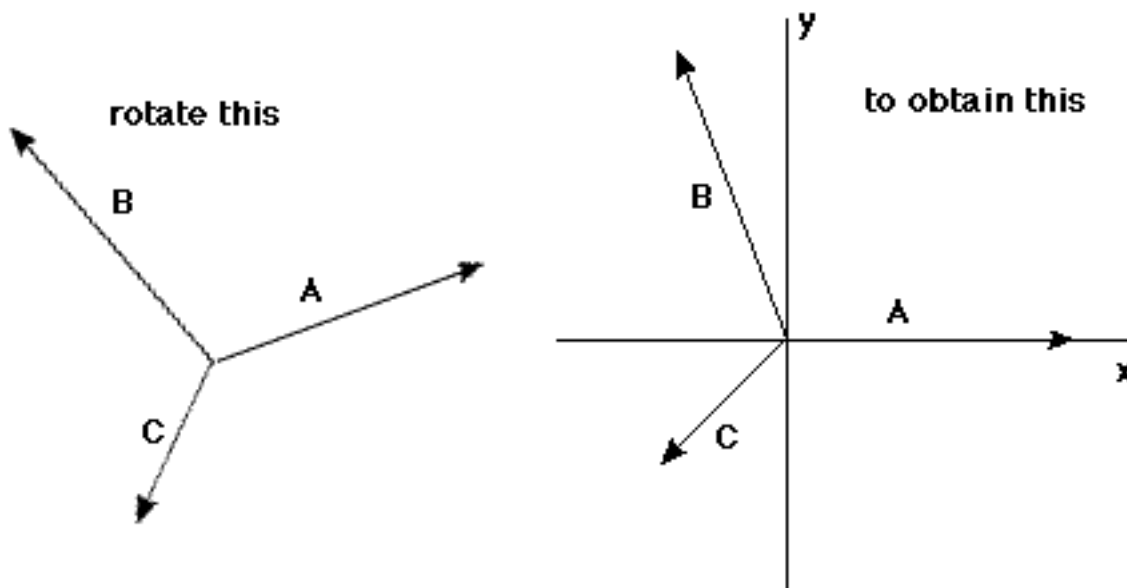
Your calculator returns the value of  $-81.8^\circ$ . However, it's obvious that the resultant cannot be in the 4th quadrant. There is a 2nd solution to this problem: the other angle is  $\theta + 180^\circ = 98.2^\circ$ , an angle that is in the 2nd quadrant.

The resultant vector appears below.



The above angle could also have been expressed as  $81.8^\circ$  from the negative side of the x-axis.

When you are resolving more than two vectors (as on a force table), it is easiest if you choose your coordinate system so that one of the vectors is on the positive x-axis ( $\theta = 0^\circ$ ). See the example below.



As far as measuring angles goes, you have two options:

- 1) measure the smallest angle from the x-axis and use symmetry rules to keep track of signs; e.g.,  $B_x$  is (-) and both  $C_x$  and  $C_y$  are (-), or
- 2) always use standard measures of angles when performing trig functions; e.g., angle B is roughly  $115^\circ$  and angle C is nearly  $225^\circ$ .

In either case, you will have to draw upon what you learned in math class to resolve vectors into x and y components and find the resultant.